PRELIMINARY COMPUTATIONS ON THE MAINTENANCE AND PREDICTION OF SEASONAL TEMPERATURES IN THE TROPOSPHERE [1]

JULIAN ADEM [2]

Extended Forecast Branch, NMC, U.S. Weather Bureau, Washington, D.C.

[Manuscript received April 17, 1963; revised June 27, 1963]

ABSTRACT

By considering the conservation of energy and the balance of radiation in the troposphere, in the cloud layer, and in the surface of the earth, we obtain a system of six equations containing six unknowns. The variables are the temperatures and the excesses of radiation in the troposphere, in the cloud layer, and in the surface of the earth. The equations contain as parameters the insolation, the cloudiness, the heat given off by the oceans and the continents to the atmosphere, and the initial temperature distributions. The model contains only meridional turbulent transport in the troposphere. Computations for both hemispheres and for the four seasons, as well as for the annual case, are carried out, obtaining a remarkable agreement with observations.

The meridional transport is accomplished by the cyclones and anticyclones of the middle latitudes with an austausch coefficient of the order of magnitude of 5×10^{10} cm.^{2} sec.$^{-1}$.

The energy received from the surface of the earth by the atmosphere is very important for the maintenance and prediction of temperatures.

In the seasonal and in the annual cases there is no substantial storage of energy in the troposphere and a balance almost exists among the excess of radiation, the energy transported meridionally by turbulence, and the heat given off by the oceans and the continents. Therefore the prediction of the seasonal temperatures does not depend strongly on the initial temperatures in the troposphere itself.

In the annual case there also exists a balance in the upper layer of continents and oceans between the excess of radiation and the heat given off to the atmosphere. However in the seasonal case there exists a large storage of energy in the oceans and no such balance exists. Therefore, the mean seasonal temperature in the troposphere depends very strongly on the temperature in the oceans and we need to prescribe the temperature in the oceans at the early part of the season to predict the mean tropospheric temperature for the whole season.

1. INTRODUCTION

In a previous paper [1] by considering the incoming and outgoing radiation in the troposphere, in the cloud layer, and in the surface of the earth, we derived three equations that have as unknowns the temperature and the excess of radiation in the mid-troposphere, in the cloud layer, and in the surface of the earth. The known parameters in the equations are the insolation and the cloud cover. Furthermore, we derived the equation of conservation of energy for the troposphere.

By applying the principle of conservation of energy at the surface of the earth and at the cloud layer, we get two more equations that, combined with the four previous ones, allow us to determine the six unknowns. In this way we obtain the temperature distributions in the troposphere and in the surface of the earth as functions of the insolation, the cloudiness, the energy given off by the surface of the earth to the atmosphere (as latent and sensible heat), and of the initial temperature distribution.

[1] This research was started at the Institut für Meereskunde der Universität Hamburg, while the author was a visiting professor during the winter 1961–62

[2] On leave from Universidad Nacional de México.

2. THE EQUATION OF CONSERVATION OF ENERGY IN THE UPPER LAYER OF THE OCEANS AND CONTINENTS

We shall assume that the temperature in the oceans and continents at a depth h is constant and that in the upper layer of thickness h the mean temperature is given by $(T_s+T_h)/2$, where T_s is the temperature at the surface, and T_h the temperature at the depth h. We shall take $T_s=T_{s_0}+T'_s$ where T_{s_0} is a constant and $T'_s<<T_{s_0}$.

With the above assumptions the energy equation for the upper layer of thickness h in the oceans and continents can be written as

$$\frac{h\rho c}{2}\frac{\partial T'_s}{\partial t}=E_s-G+R \qquad (1)$$

where ρ is the density, c is the specific heat, E_s is the excess of radiation per unit time and unit area, G is the total energy given off to the atmosphere per unit time and unit area (latent and sensible heat); R includes all other terms and will be neglected.

In a way similar to that in the previous paper [1] we integrate equation (1) with respect to the time t and with respect to the longitude ψ, in order to obtain the equation that governs the mean values of the variables over an interval of time Δt and over the circles of latitude ϕ.

Let L be the ratio of the total length of ocean to the length of the whole circle corresponding to latitude ϕ, then $1-L$ is the ratio of the length of the continent at the same latitude.

After the averaging we get the following relations:

$$\begin{aligned} \overline{T}'_s &= L\overline{T}'_w + (1-L)\overline{T}'_e \\ \overline{G} &= L\overline{G}_w + (1-L)\overline{G}_e \end{aligned} \tag{2}$$

where $\overline{T}'_s$ and $\overline{G}$ are the mean values over the whole circle, $\overline{T}'_w$ and $\overline{G}_w$ are the mean values on the oceans, and $\overline{T}'_e$ and $\overline{G}_e$ are the mean values on the continents. The time averages are taken for the Δt interval.

The equation obtained for the surface of the earth after the averaging is the following:

$$-L\frac{h_w\rho_w c_w}{2}\frac{T'_{w_1}-T'_{w_n}}{\Delta t}-(1-L)\frac{h_e\rho_e c_e}{2}\frac{T'_{e_1}-T'_{e_n}}{\Delta t} = \overline{E}_s - L\overline{G}_w - (1-L)\overline{G}_e \tag{3}$$

where the subscripts w and e indicate values in the oceans and in the continents, respectively. T'_{w_1} and T'_{w_n} are respectively the mean surface temperatures in the ocean at the beginning and at the end of the considered time interval and T'_{e_1} and T'_{e_n} are the mean surface temperature on the continent at the beginning and at the end of the same interval. These temperatures are not time averages.

We shall divide the interval Δt in n subintervals of equal length $\Delta t/n$. To use the mean values on the n subintervals as data, we shall make the following approximation

$$\begin{aligned} \frac{T'_{w_1}-T'_{w_n}}{n} &= \frac{\overline{T}'_{w_1}-\overline{T}'_{w_n}}{n-1} \\ \frac{T'_{e_1}-T'_{e_n}}{n} &= \frac{\overline{T}'_{e_1}-\overline{T}'_{e_n}}{n-1} \end{aligned} \tag{4}$$

where $\overline{T}'_{w_n}$ and $\overline{T}'_{e_n}$ are the mean temperatures during the last subinterval and $\overline{T}'_{w_1}$ and $\overline{T}'_{e_1}$ the mean temperatures at the first subinterval. By choosing sufficiently small subintervals, relations (4) become as approximate as desired.

The mean temperature in the interval Δt can be written as

$$\overline{T}'_w = \frac{1}{n}\sum_{l=1}^{n}\overline{T}'_{w_l}$$

where $\overline{T}'_{w_l}$ is the mean value in each of the n subintervals of the length $\Delta t/n$ in which Δt has been divided.

Therefore,

$$\overline{T}'_{w_n} = n\overline{T}'_w - \sum_{l=1}^{n-1}\overline{T}'_{w_l}$$

and we get

$$T'_{w_1}-T'_{w_n} = -\frac{n^2}{n-1}\left[\overline{T}'_w - \left(\overline{T}'_{w_1}+\sum_{l=1}^{n-1}\overline{T}'_{w_l}\right)/n\right] \tag{5}$$

Similarly, we get

$$T'_{e_1}-T'_{e_n} = -\frac{n^2}{n-1}\left[\overline{T}'_e - \left(\overline{T}'_{e_1}+\sum_{l=1}^{n-1}\overline{T}'_{e_l}\right)/n\right] \tag{6}$$

Substituting (5) and (6) in equation (3) we obtain

$$Ld_w[\overline{T}'_w-(\overline{T}'_w)_i]+(1-L)d_e[\overline{T}'_e-(\overline{T}'_e)_i] = \overline{E}_s - L\overline{G}_w-(1-L)\overline{G}_e \tag{7}$$

where $\overline{T}'_w + \overline{T}'_e = f(t_s)$

$d_w=(n^2/n-1)\,h_w\rho_w c_w/2\Delta t$ and $d_e=(n^2/n-1)\,h_e\rho_e c_e/2\Delta t$.

The values of $(\overline{T}'_w)_i$ and $(\overline{T}'_e)_i$ are given by:

$$\begin{aligned} (\overline{T}'_w)_i &= (\overline{T}'_{w_1}+\sum_{l=1}^{n-1}\overline{T}'_{w_l})/n \\ (\overline{T}'_e)_i &= (\overline{T}'_{e_1}+\sum_{l=1}^{n-1}\overline{T}'_{e_l})/n \end{aligned} \tag{8}$$

3. THE MATHEMATICAL MODEL AND ITS SOLUTION

We will use the following notation: $T_m=T_{m_0}+T'_m$ is the mean temperature in the troposphere, (or the temperatures at 5.5 km. above sea level) where T_{m_0} is a constant and $T'_m \ll T_{m_0}$; $T_c=T_{c_0}+T'_c$ is the temperature at the cloud cover, where T_{c_0} is a constant and $T'_c \ll T_{c_0}$; E_A is the excess of radiation in the troposphere (excluding the cloud layer) and E_s the excess of radiation at the surface of the earth.

We shall use the equation of conservation of energy for the troposphere in the form given by the author [1]. We apply to it the averaging operator

$$\frac{1}{2\pi\Delta t}\int_0^{2\pi}d\psi\int_t^{t+\Delta t}(\ )dt$$

and introduce the approximation

$$T'_{m_1}-T'_{m_n} = -\frac{n^2}{n-1}[\overline{T}'_m-(\overline{T}'_m)_i] \tag{9}$$

where T'_{m_1} and T'_{m_n} are the mean temperatures at the beginning and at the end of the interval and $\overline{T}'_m$ is the mean temperature on the whole interval.

The values of $(\overline{T}'_m)_i$ are computed from

$$(\overline{T}'_m)_i = (\overline{T}'_{m_1}+\sum_{l=1}^{n-1}\overline{T}'_{m_l})/n \tag{10}$$

where $\overline{T}'_{m_l}$ is the mean temperature in each of the n equal subintervals into which Δt has been divided.

After the averaging, the equation of conservation of energy for the troposphere (equation (20) of reference [1]) becomes:

$$c_9[\overline{T}'_m-(\overline{T}'_m)_i]+c'_7\overline{K}\left(\frac{d^2\overline{T}'_m}{d\phi^2}-\tan\phi\,\frac{d\overline{T}'_m}{d\phi}\right)=\overline{E}_A+L\overline{G}_w+(1-L)\overline{G}_e \quad (11)$$

$\overline{T}'_m = f(\ T',\ )$

where $c_9=(n^2/n-1)r_0^2c'_7/\Delta t$; $c'_7=c_7/c_3$; c_3 and c_7 are constants.

In this equation, the term $c_9[\overline{T}'_m-(\overline{T}'_m)_i]$ represents the storage of thermal energy; the term $c'_7\overline{K}(d^2\overline{T}'_m/d\phi^2-\tan\phi\, d\overline{T}'_m/d\phi)$ is the rate of change of thermal energy due to the horizontal turbulent transport; $\overline{E}_A$ is the thermal energy gained by radiation.

The terms $L\overline{G}_w$ and $(1-L)\overline{G}_e$ in equation (11) represent the rate of change of thermal energy in the troposphere due to turbulent vertical conduction at the earth's surface and condensation in the clouds, and since their value has been taken as the negative of the last two terms of the second member of (7) we have assumed that the energy released by the surface of the earth at a given latitude is gained in the troposphere as sensible heat at the same latitude and time, which implies that at a given latitude the energy released by evaporation at the surface of the earth is equal to the energy released by the condensation of water vapor in the clouds. This assumption will increase slightly the required austausch coefficient $\overline{K}$. However, at least for the middle latitudes, this approximation will show in a crude way if the energy given off to the troposphere from the surface of the earth by evaporation and by turbulent vertical transport, combined with that given off by radiation and by horizontal eddy transport is of the correct order of magnitude to maintain the observed mean temperature in the troposphere.

When the time interval Δt is taken sufficiently large, other terms omitted in equation (11) are presumably too small to be included in these preliminary computations, and therefore will be neglected.

The conservation of energy condition at the cloud layer can be replaced by the condition that the temperature at the cloud layer is equal to the temperature of the atmosphere at the same height. Therefore, by choosing appropriately T_{c_0} and T_{m_0} we get the condition

$$\overline{T}'_m=\overline{T}'_c \quad (12)$$

Finally $\overline{T}'_m$, $\overline{T}'_c$, $\overline{T}'_w$, $\overline{T}'_e$ are related to $\overline{E}_A$, $\overline{E}_c$, and $\overline{E}_w$ by the equations of radiation balance derived in a previous paper [1], which are the following:

$$c_8\overline{T}'_m=c_0+c_1\overline{I}+c_3\overline{E}_A+c_4\overline{E}_s+c_5\overline{E}_c \quad (13)$$

$$-b_2A\overline{T}'_m+L\overline{T}'_w+(1-L)\overline{T}'_e=b_0+b_1\overline{I}+b_3\overline{E}_A+b_4\overline{E}_s+b_5\overline{E}_c \quad (14)$$

$\overline{T}'_m = f(\ ,\ \overline{E}_A,\ \overline{E}_s,\ \overline{E}_c)$

$\overline{T}'_m\ \overline{T}'_w\ \overline{T}'_c = f(\ ,\ \overline{E}_A,\ \overline{E}_s,\ \overline{E}_c)$

$$-d_2A\overline{T}'_m+\overline{T}'_c=d_0+d_1\overline{I}+d_3\overline{E}_A+d_4\overline{E}_s+d_5\overline{E}_c \quad (15)$$

where A, c_3, c_i, b_i, and d_i (for $i=0, 1, 2, 3, 4, 5$) are constants.

If in the system (7), (11), (12), (13), (14), and (15) we take as unknowns the temperatures $\overline{T}'_m$, $\overline{T}'_w$, $\overline{T}'_e$, and $\overline{T}'_c$ and the excesses of radiation $\overline{E}_A$, $\overline{E}_c$, and $\overline{E}_s$, we need an extra equation to get the solution. However, since in equation (7) $d_e<<d_w$, when t is sufficiently large, we can eliminate $\overline{T}'_e$ from this equation and get an approximate solution of the system replacing $L\ \overline{T}'_w+(1-L)\ \overline{T}'_e$ by $\overline{T}'_w$ in equation (14). This approximate solution is equivalent to the exact solution of the problem when $L=1$, except that in equation (7), instead of the coefficient d_w, $L\ d_w$ appears. In other words, the approximate solution corresponds to the case of an earth completely covered by an ocean in which the value of h_w is taken L times smaller, and the value of G_w is replaced by $L\ \overline{G}_w+(1-L)\ \overline{G}_e$.

Taking $L=1$ and using equations (7), (12), (13), (14), and (15) we obtain

$$\overline{E}_A=D_{11}\overline{T}'_m+D_{21}\overline{I}+D_{31}\overline{G}+D_{41}d_w(\overline{T}'_s)_i+D_{51} \quad (16)$$

$$\overline{T}'_s=D_{12}\overline{T}'_m+D_{22}\overline{I}+D_{32}\overline{G}+D_{42}d_w(\overline{T}'_s)_i+D_{52} \quad (17)$$

$$E_c=D_{13}\overline{T}'_m+D_{23}\overline{I}+D_{33}\overline{G}+D_{43}d_w(\overline{T}'_s)_i+D_{53} \quad (18)$$

where

$$D_{1k}=[c_8\Delta_{1k}-b_2A\Delta_{2k}+(1-d_2A)\Delta_{3k}]/\Delta,$$

$$D_{2k}=-(c_1\Delta_{1k}+b_1\Delta_{2k}+d_1\Delta_{3k})/\Delta,$$

$$D_{3k}=-(c_4\Delta_{1k}+b_4\Delta_{2k}+d_4\Delta_{3k})/\Delta,$$

$$D_{4k}=-D_{3k},$$

$$D_{5k}=-(c_0\Delta_{1k}+b_0\Delta_{2k}+d_0\Delta_{3k})/\Delta$$

where

$$k=1,2,3 \text{ and } \Delta_{11}=d_w(b_4d_5-b_5d_4)-d_5;$$

$$\Delta_{21}=d_w(d_4c_5-d_5c_4);\ \Delta_{31}=d_w(c_4b_5-b_4c_5)+c_5;$$

$$\Delta_{12}=d_3b_5-b_3d_5;\ \Delta_{22}=c_3d_5-d_3c_5;$$

$$\Delta_{32}=b_3c_5-b_5c_3;\ \Delta_{13}=d_w(b_3d_4-b_4d_3)+d_3;$$

$$\Delta_{23}=d_w(d_3c_4-d_4c_3);\ \Delta_{33}=d_w(c_3b_4-c_4b_3)-c_3;$$

$$\text{and } \Delta=c_3\Delta_{11}+b_3\Delta_{21}+d_3\Delta_{31}.$$

Therefore, the variables $\overline{E}_A$, $\overline{E}_c$, $\overline{E}_s$, $\overline{T}'_s$, and $\overline{T}'_c$ are completely determined if we know $\overline{T}'_m$.

Substituting (16) in (11) we obtain the equation that governs $\overline{T}'_m$:

$$c'_7\overline{K}\left(\frac{d^2\overline{T}'_m}{d\phi^2}-\tan\phi\,\frac{d\overline{T}'_m}{d\phi}\right)-c'_9\overline{T}'_m=F'(\phi) \quad (19)$$

where $F'(\phi)=D_{21}\overline{I}+(1+D_{31})\overline{G}+d_wD_{41}(\overline{T}'_s)_i+c'_9(\overline{T}'_m)_i+D_{51}$ and $c'_9=-c_9+D_{11}$.

Equation (19) is a generalization of a previous one obtained by the author [1]. The forcing function $F'(\phi)$ linearly contains the insolation $(\overline{I})$, the heat given off from the surface of the earth to the atmosphere (by evaporation and by conduction of sensible heat) ($\overline{G}$), and the temperature field at the early part of the considered time interval $((\overline{T}'_s)_i$ and $(\overline{T}'_m)_i)$.

If we neglect the term $-c'_7\overline{K}\tan\phi(dT'_m/d\phi)$ the solution of (19) for $K\neq 0$ is

$$\overline{T}'_m=-\frac{1}{2c'_6c'_7\overline{K}}\left[e^{-c'_6\phi}\int F'(\phi)e^{c'_6\phi}d\phi-e^{c'_6\phi}\int F'(\phi)e^{-c'_6\phi}d\phi\right]+K_1e^{-c'_6\phi}+K_2e^{c'_6\phi} \quad (20)$$

where $c'_6=(c'_9/c'_7\overline{K})^{1/2}$ and the arbitrary constants K_1 and K_2 are determined from the conditions that $(dT'_m/d\phi)_{\phi=0}=0$ and $(dT'_m/d\phi)_{\phi=\pi/2}=0$.

For $\overline{K}=0$ the solution is

$$\overline{T}'_m=-\frac{F'(\phi)}{c'_9} \quad (20')$$

The mathematical model here presented and whose solution is given by (20), (16), (17), (18), (7), and (12) contains meridional transport only in the troposphere, through the turbulent transport by horizontal eddies, represented by the austausch coefficient $\overline{K}$.

The omission of the term $-c'_7\overline{K}\tan\phi(d\overline{T}'_m/d\phi)$ in equation (19) will increase slightly the required austausch coefficient, $\overline{K}$, as well as also change slightly the computed temperature, $\overline{T}'_m$. However, since the dominant transport term is $c'_7\overline{K}(d^2\overline{T}'_m/d\phi^2)$, the solution will be substantially correct, except when we approach to the Pole.

The solution also depends on the cloud cover, which is a parameter included in the coefficients.

4. MAINTENANCE AND PREDICTION OF THE TEMPERATURES IN THE CASES OF THE FOUR SEASONS OF THE YEAR

THE NORTHERN HEMISPHERE

We shall investigate how the mean temperature distribution of a season is maintained and how it is affected by the temperature distribution of the early part of the season. For this purpose we shall use the values of heat given off by the ocean to the atmosphere (by evaporation and by conduction of sensible heat) given by Jacobs [4]. Let G_p and G_A be the average of the values for the North Pacific and the North Atlantic, respectively, then if L_p and L_A are the lengths in the Pacific and in the Atlantic of the 10° width zones to which G_p and G_A correspond respectively then $\overline{G}_w=(G_pL_p+G_AL_A)/(L_p+L_A)$ is the mean value of the heat given off by the oceans to the atmosphere in the Northern Hemisphere.

The values of $\overline{G}_w$ are given in figure 1. The curves labeled I, III, IV, and II correspond to the average values of $\overline{G}_w$, for the 3-month periods December-January-February, March-April-May, June-July-August, and September-October-November, respectively. The 3-month periods chosen do not exactly correspond to the four seasons of the year; however, we divided the year in this way in order to use the values given by Jacobs [4].

The values of $\overline{I}$ for the same periods were computed from the Milankovitch formula [1].

The values of $(T'_m)_i$ were taken from the data published by the U.S. Weather Bureau [10], by reducing the values given there for 700 mb. to values for 500 mb. The values of surface temperatures for the North Atlantic were taken from Defant [3] and the average values of the temperature at the surface of the earth for the whole Northern Hemisphere from Landsberg [5].

To evaluate $(T'_m)_i$ and $(T'_s)_i$ we shall use subintervals of a month. Therefore in formulas (8) and (10) we will take $n=3$.

We shall consider the case in which the sky is covered by a 50 percent cloud cover. The constants that appear in the solution were already computed for this case by the author [1] and will be taken from there.

The only new constants that we have to add are the following:

$$\rho_w=1 \text{ gm. cm.}^{-3} \text{ and } c_w=1 \text{ cal. gm.}^{-1}.$$

The results of the computations are shown in figures 2, 3, and 4. The curves labeled with I and II correspond to the solution with $\overline{G}_w=\overline{G}_e=(G_w)_{\text{Jacobs}}$, $\overline{K}=5\times10^{10}$ cm.^{2} sec.$^{-1}$, and $Lh_w=10^4$ cm. In case I we have computed $(\overline{T}'_s)_i$ from the mean monthly surface temperatures in the North Atlantic and, in case II, from the mean monthly surface temperatures for the whole Northern Hemisphere.

Curves III correspond to the same case as II but with $Lh=5\times10^3$ cm.

Curves IV and V correspond to the same case as III, except that in IV we have used $\overline{G}_e=0$ and in V, $\overline{G}_w=0$ and $\overline{G}_e=0$.

Curves VI correspond to the same case as III, except that in this case we have used $\overline{K}=0$.

In figures 2, 3, and 4 the dashed lines correspond to the observed values, taken from the following sources: $\overline{T}'_m$ from the U.S. Weather Bureau [10], $\overline{T}'_s$ for the whole Northern Hemisphere from Landsberg [5], and the values of the difference between incoming and outgoing radiation, $\overline{E}_A+\overline{E}_s+\overline{E}_c$, from London [7]. The dotted lines correspond to the observed values of $\overline{T}'_s$ for the North Altantic and are taken from Defant [3].

THE SOUTHERN HEMISPHERE

Since the Southern Hemisphere has a negligible percentage of continents, our model of an earth completely covered by an ocean should give much better results in this case than when applied to the Northern Hemisphere. To compute this case we shall take as initial temperature distributions the temperatures at 500 mb. and at sea level given by the Soviet Ministry of Meteorology [8]. However, as values of the energy given off by the ocean to the atmosphere, we shall use the Northern Hemisphere data given by Jacobs [4].

We will take $h_w=100$ m., $\overline{G}_w=(\overline{G}_w)_{\text{Jacobs}}$, $L=1$, $\overline{G}_e=0$, and $\overline{K}=5\times10^{10}$ cm.^{2} sec.$^{-1}$

The results of the computations are shown in figures 5, 6, 7. The thin line for the summer case corresponds to $\overline{K}=3\times10^{10}$ cm.^{2} sec.$^{-1}$

5. GENERAL RESULTS AND CONCLUSIONS

Comparison of curves I, II, and III with the dashed and dotted curves shows that the solution for both hemispheres agrees remarkably well with observations. The best agreement is obtained in the transition seasons (spring and autumn). The solution for summer is colder than the observed value, while the solution for winter is warmer. This is due to the hypothesis of replacing the continents by an ocean.

Curves V, which correspond to the hypothetical case in which no energy is given off from the total surface of the earth by evaporation or by heat conduction, show the importance of this energy in maintaining the atmospheric temperature. In fact, in the absence of this energy the temperature would be considerably lower in the atmosphere. At 20° latitude, it would be 14° K., 12° K., 12° K., and 9° K. lower, in the winter, autumn, spring, and summer respectively; at 40° latitude it would be 9° K., 9° K., 8° K., and 8° K., and at 60° latitude it would be 7° K., 5° K., 4° K., and 3° K. lower.

From these results it is clear that evaporation and conduction of sensible heat from the surface play an important role in maintaining the atmospheric temperature.

Finally, curve VI corresponds to the solution in which no meridional transport exists. Its comparison with I shows that the meridional transport is accomplished horizontally by the large cyclones and anticyclones of the middle latitudes that correspond to an exchange of coefficients $\overline{K}$ of the order of magnitude of 5×10^5 cm.^{2} sec.$^{-1}$, in agreement with previous work [2, 6, 1].

As anticipated by the author [1] the introduction in the computation of the observed value of $\overline{G}$ for the summer corrects the unrealistic solution obtained before on the assumption of a constant $\overline{G}$. In the other seasons the gradient of radiant energy is considerably larger than that of $\overline{G}$, and for this reason the solution given before, using $\overline{G}$ equal to a constant, was in agreement with the observations.

Figure 4 shows that the excesses of radiation in the troposphere ($\overline{E}_A$) and in the cloud layer ($\overline{E}_c$) are negative for all seasons. This means that a deficit of radiation in the troposphere exists during the whole year.

The sum of the excesses of radiation ($\overline{E}_A+\overline{E}_c+\overline{E}_s$) is equal to the difference between the incoming and the outgoing radiation at the top of the troposphere and agrees remarkably well with the values given by London [7].

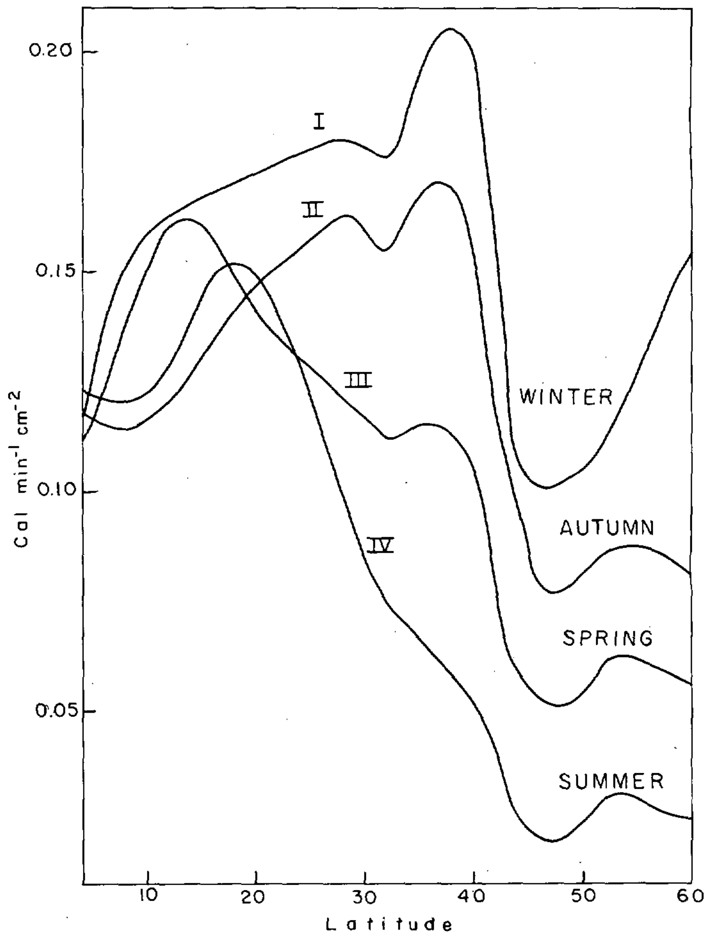


Figure 1.—Energy given off by the oceans to the atmosphere in the Northern Hemisphere.

Although for the case of the Northern Hemisphere the best solution is obtained when using the mean temperature at the surface as initial value, the best solution for $\overline{E}_s$ is obtained when the initial temperature is the mean temperature of the ocean; this is due to the fact that $\overline{E}_s$ is determined from equation (7) in which $d_e<<d_w$.

If in the computations we neglect the term c_9 $(\overline{T}'_m-(\overline{T}'_m)_i)$ of equation (11), the results are the same as those obtained above, except in autumn in which the computed tropospheric temperature becomes about 1° lower. This result shows that in the 3-month intervals there is very little storage of energy in the troposphere and a balance almost exists among the excess of radiation, the meridional transport by turbulence, and the heat received from the surface of the earth (by evaporation and conduction). Therefore the prediction of the seasonal temperatures does not depend strongly on the initial temperatures in the troposphere itself.

The neglect in the computations of $L\ d_w\ (T'_s-(\overline{T}'_s)_i)$ produces curve VII showing that in the 3-month intervals there exists a large storage of energy in the oceans.

However, if we perform the computations for an interval

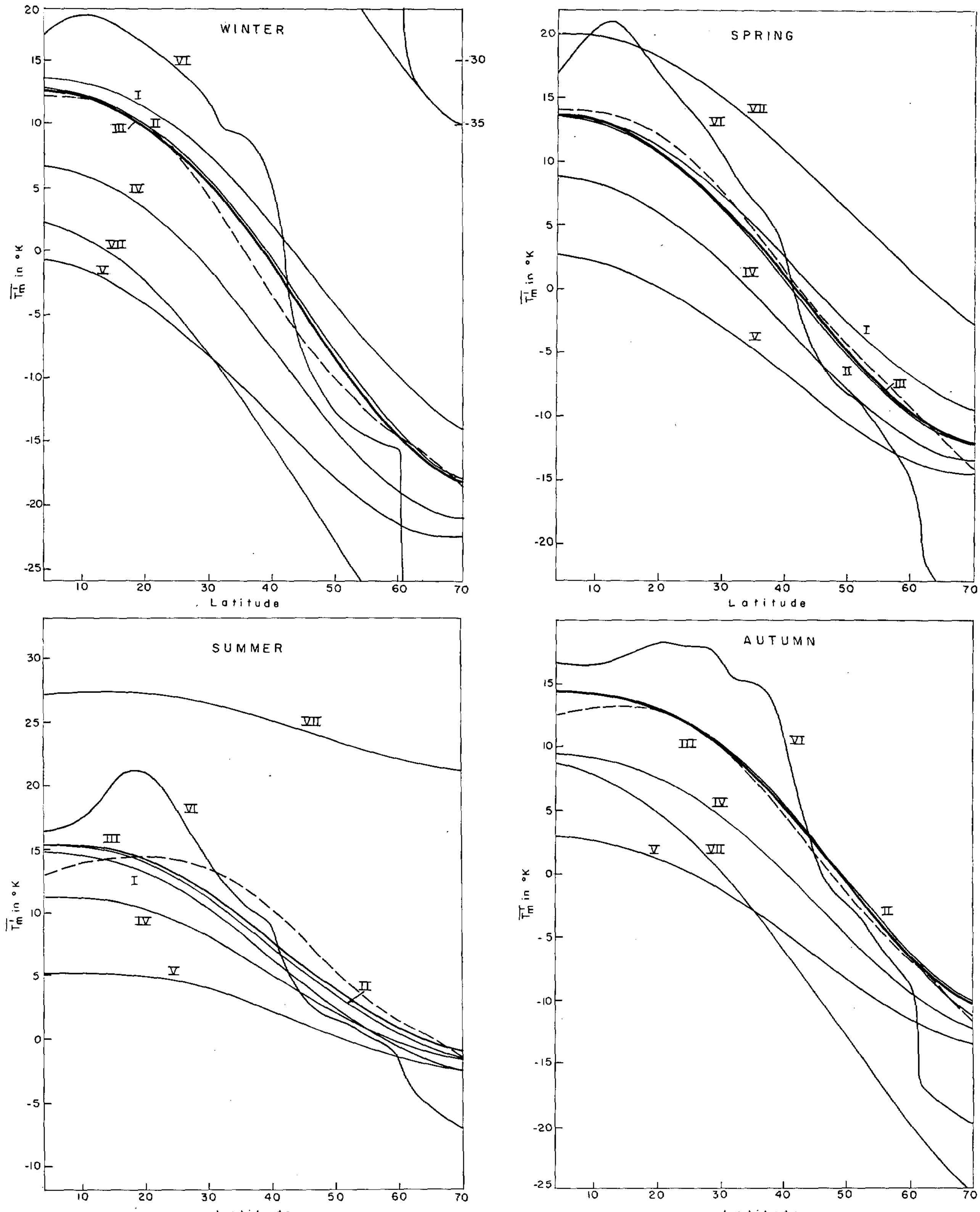


FIGURE 2.—Mean temperature in the troposphere for the Northern Hemisphere by seasons. Dashed curve is the observed temperature for total surface of earth; curve I is computed using $\overline{G}_w=\overline{G}_e=(G_w)_{\text{Jacobs}}$, $\overline{K}=5\times10^{10}$ cm.^{2} sec.$^{-1}$, $Lh_w=10^4$ cm., the mean temperature in the ocean as initial conditions; curve II is same as curve I but with the mean temperature on total surface of earth as initial condition; curve III is same as curve II but with $Lh=5\times10^3$ cm.; curve IV is same as III except with $\overline{G}_e=0$; curve V is same as III except with $\overline{G}_w=0$ and $\overline{G}_e=0$; curve VI is same as III with $\overline{K}=0$; curve VII results when $Ld_w(\overline{T'}_s-(\overline{T'}_s)_i)$ is neglected.

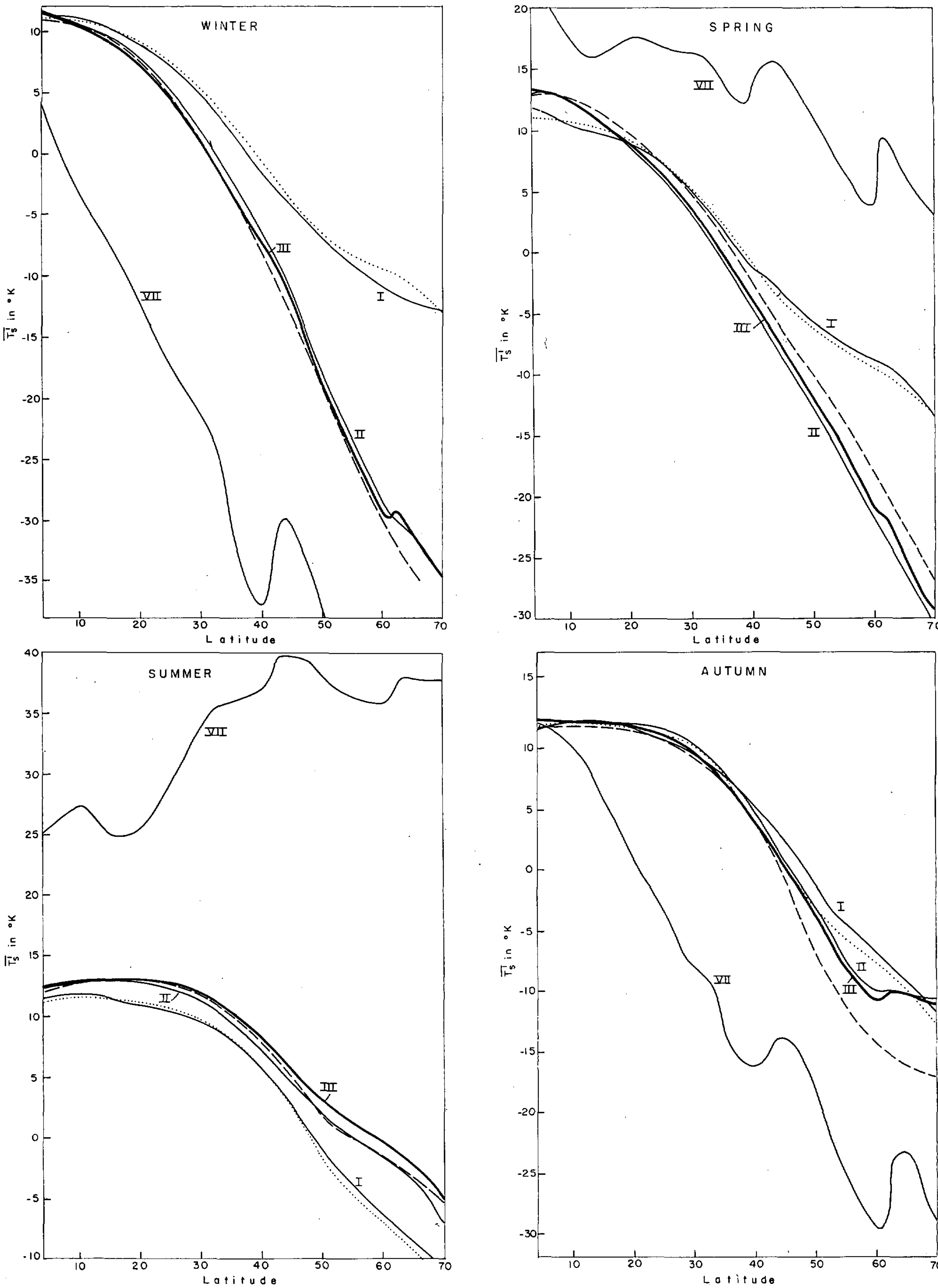


FIGURE 3.—The dashed curve is the mean observed temperature in the total surface of the earth and the dotted one the mean temperature in the ocean, for the Northern Hemisphere. Curves III and I are the corresponding computed temperatures. Curve II is same as III with $Lh_w=10^4$ cm. and curve VII neglects the term $Ld_w(\overline{T'}_s-(\overline{T'}_s)_i)$.

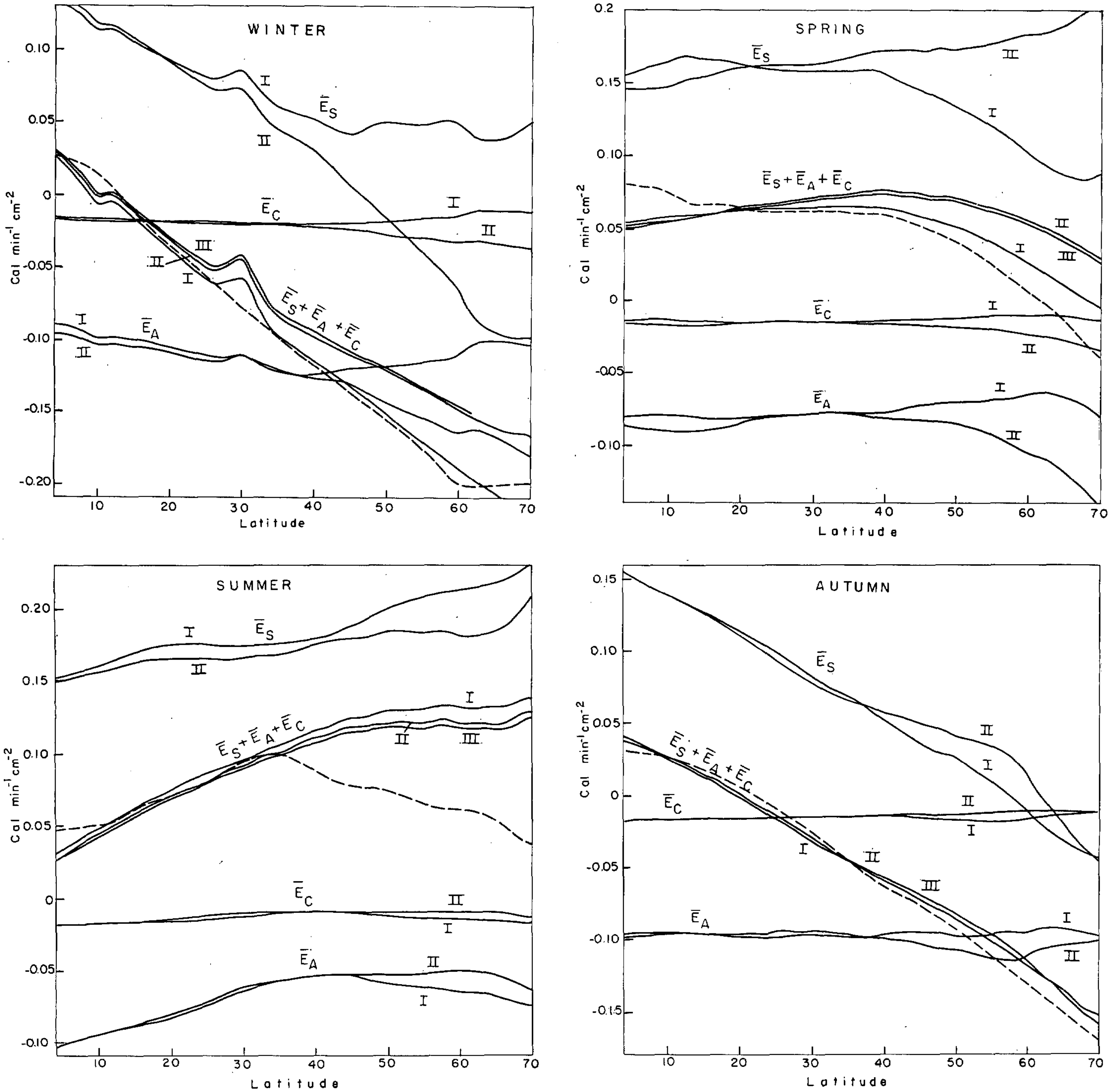


FIGURE 4.—Theoretically computed excess of radiation in the Northern Hemisphere. $\overline{E}_A$ is the excess in the troposphere, $\overline{E}_c$ the excess in the clouds, and $\overline{E}_s$ in the upper layer of the earth (continents and oceans). $\overline{E}_s+\overline{E}_A+\overline{E}_c$ is the computed total excess and the dashed line is the curve given by London [7]. Curves labeled as in figure 2.

of one year, the term $L\, d_w\, (T'_s-(\overline{T}'_s)_i)$ becomes negligible in equation (7) showing that in this case the storage of energy is negligible and that the excess of radiation in the ocean is equal to the heat given off to the atmosphere by the ocean, as shown previously by Sverdrup [9]. The results of the computations for the annual case are shown in figure 8, where a and b show the temperatures in the troposphere and in the surface of the earth, respectively. In this case curves I, III, and VII coincide and agree remarkably well with observations. As before, curve V corresponds to the solution in which the energy given off by the continents is neglected and curve VI to the solution in

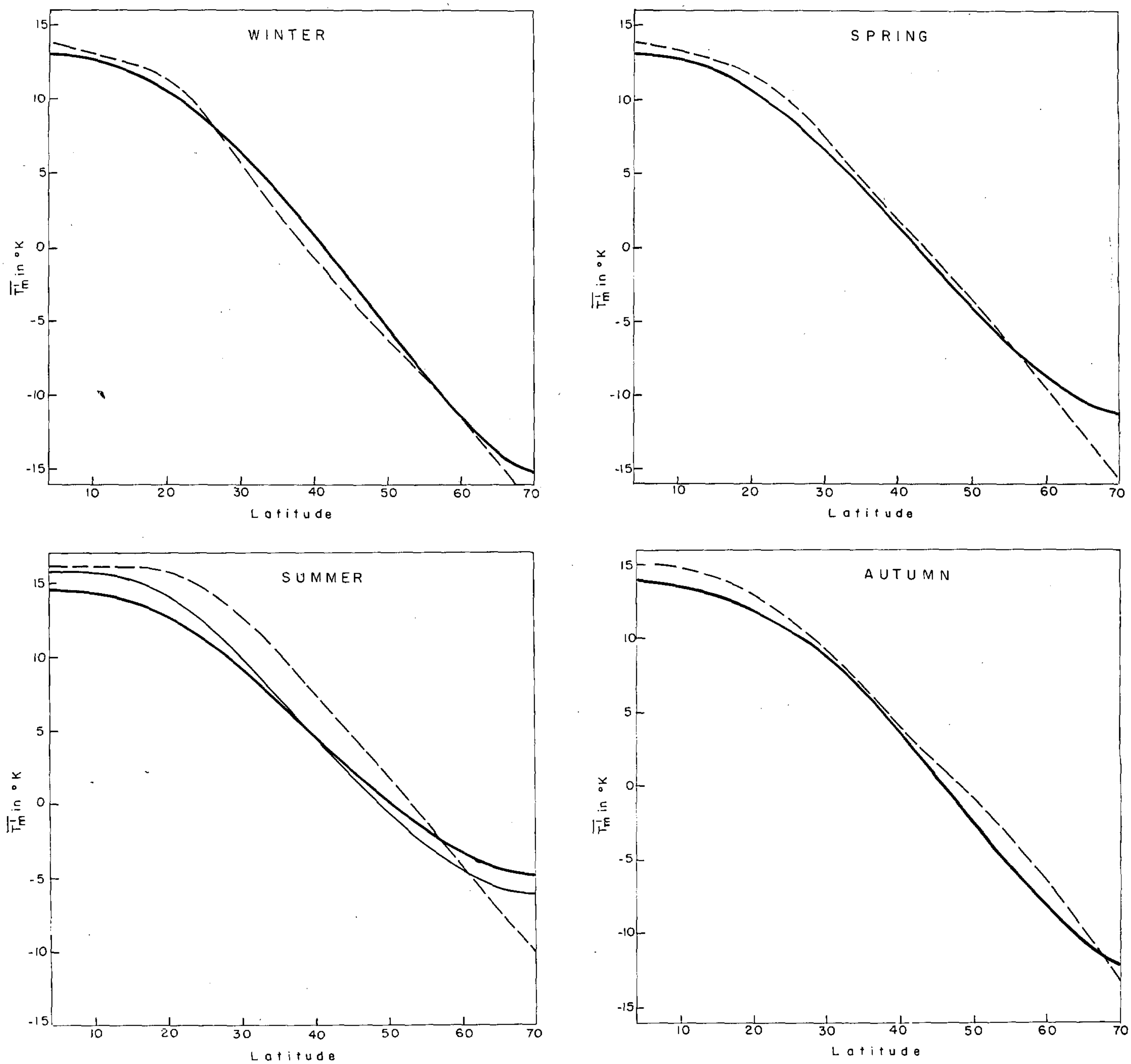


FIGURE 5.—Mean temperature during the year 1959 at the troposphere in the Southern Hemisphere. The dashed curve is observed, the solid curve computed using $h_w=100$ m., $\overline{G}_w=(\overline{G}_w)_{\text{Jacobs}}$, $L=1$, $\overline{G}_e=0$, and $\overline{K}=5\times10^{10}$ cm.^{2} sec.$^{-1}$ In the summer case the thin line corresponds to $\overline{K}=3\times10^{10}$ cm.^{2} sec.$^{-1}$

which both the energy given off by the continents and by the oceans is neglected.

Figure 8c shows the computed excesses of radiation. The annual excess of radiation in the upper layer of the earth ($\overline{E}_s$) is positive, and in the troposphere ($\overline{E}_A+\overline{E}_c$) is negative. The total excess ($\overline{E}_s+\overline{E}_A+\overline{E}_c$) agrees with the values given by London [7] which are represented by the dashed line.

The above results show that the mean seasonal temperature in the troposphere depends very strongly on the temperature in the oceans. This is clearly seen by comparing the computations of temperatures in both hemi-

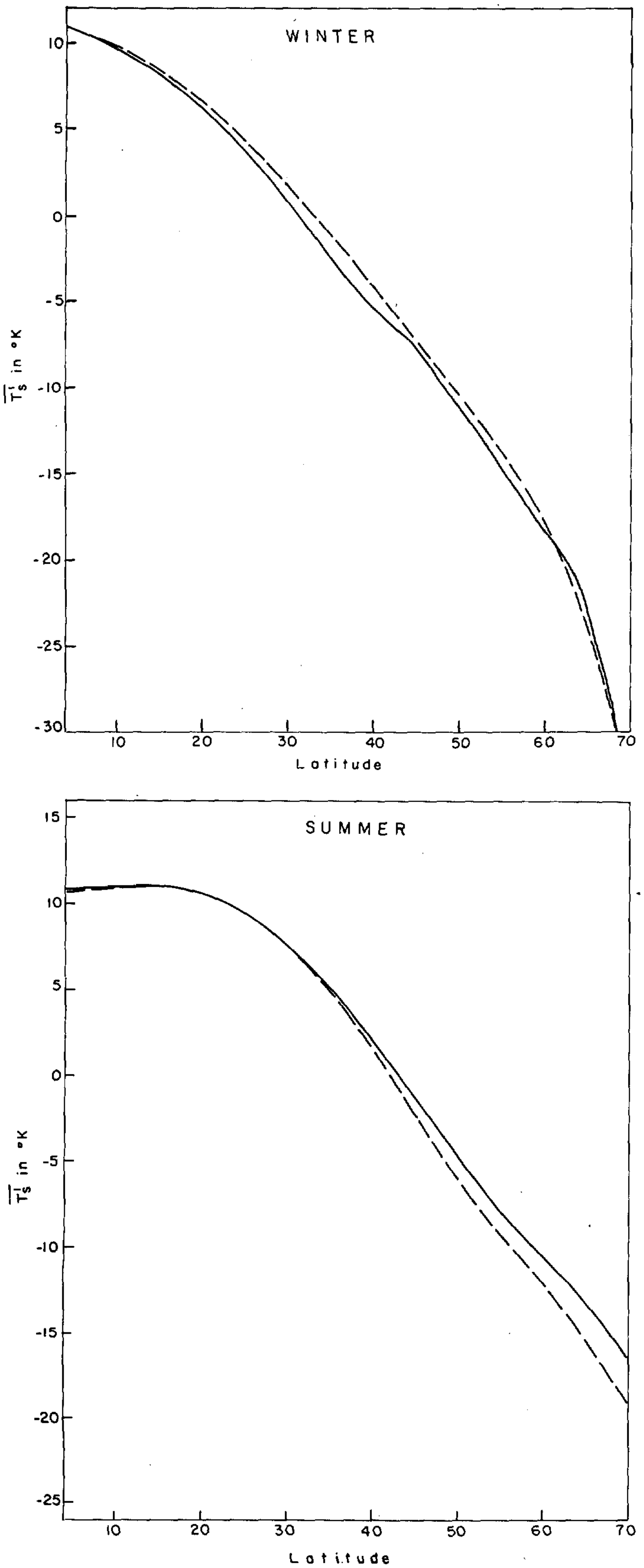


FIGURE 6.—Mean temperature during the year 1959 at the surface of the earth in the Southern Hemisphere. The dashed curve is observed and the solid curve computed as in figure 5.

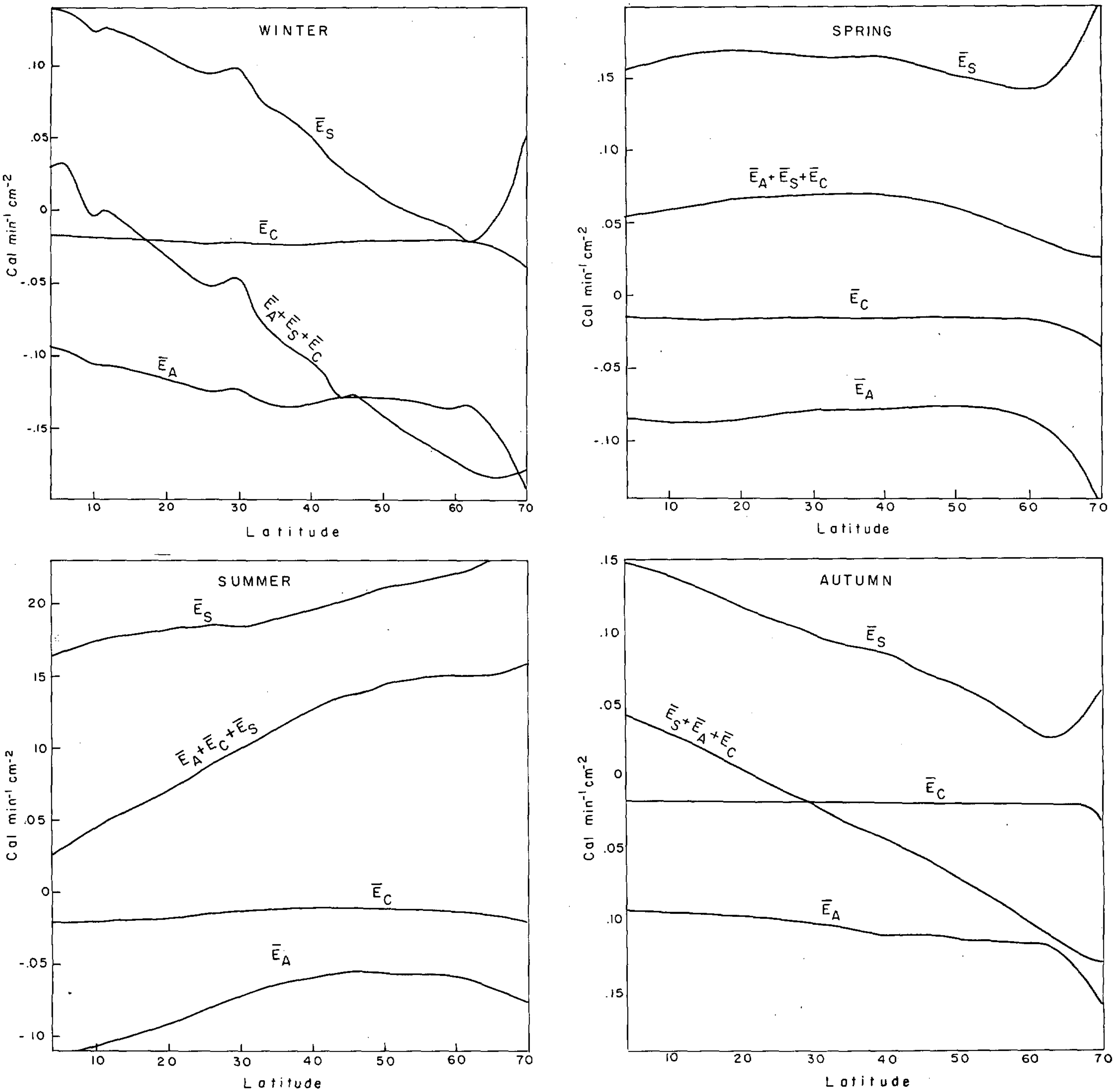
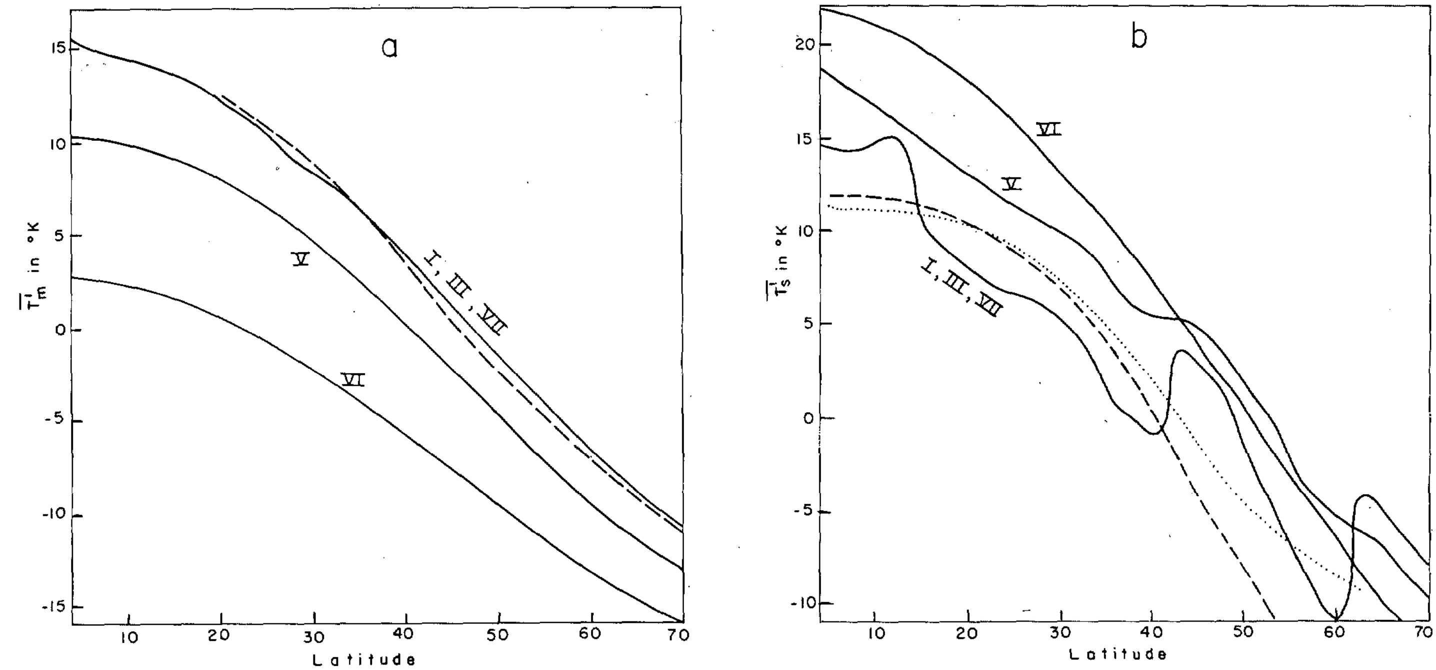


FIGURE 7.—Theoretically computed excess of radiation in the Southern Hemisphere during the year 1959. $\overline{E}_A$ is the excess in troposphere, $\overline{E}_c$ in the clouds, and $\overline{E}_s$ in the upper layer of the earth (continents and oceans).

spheres. In fact, the only essential difference in both computations is the prescribed observed temperature at the surface of the earth, and the agreement of the computed mid-tropospheric temperatures with observations is remarkably good in both hemispheres.

Therefore, for the case of a season, since the most important storage of energy is in the ocean, we need to prescribe the temperatures in the ocean at the early part of the season to predict the mean tropospheric temperature for the whole season.



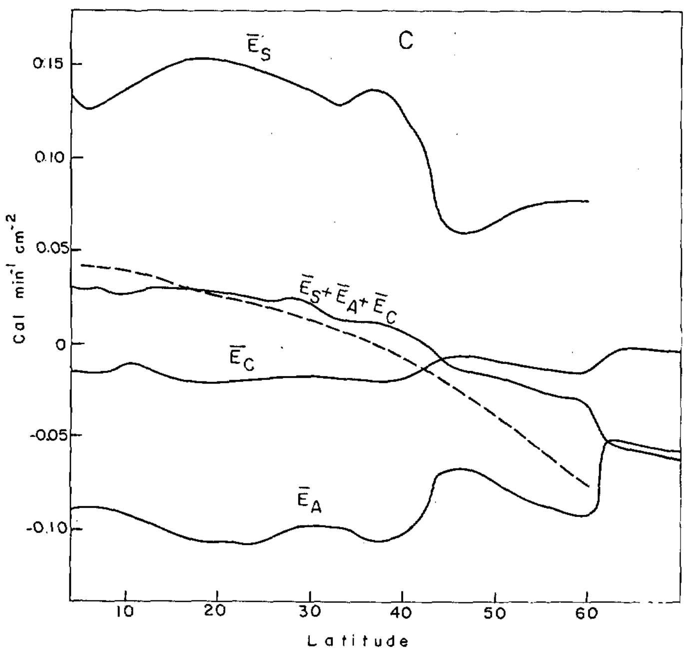


FIGURE 8.—Mean annual computed values for the Northern Hemisphere. In (a) curve I (III, VII) is the temperature in the troposphere and in (b) curve I (III, VII) is the temperature in the surface of the earth. (In the annual case the three curves coincide.) The dashed lines are the corresponding observed values. The dotted line in (b) is the observed temperature in the ocean. VI and V are the solutions, respectively when the energy given off by the total surface and by the continents is neglected. In (c) $\overline{E}_A$, $\overline{E}_c$ and $\overline{E}_s$ are the excesses of radiation in the troposphere, clouds, and upper layer of the earth, respectively. The dashed line is the total excess ($\overline{E}_A+\overline{E}_s+{}^{E_c}$), as given by London [7].

ACKNOWLEDGMENT

The author is indebted to J. Namias and P. F. Clapp for very stimulating discussions and to Gustavo Camacho and Luis Ottalengo for their help in the computations. The numerical work was carried out at the Centro Electrónico de Cálculo of the Universidad Nacional de México.

REFERENCES

1. J. Adem, "Theory of the General Circulation of the Atmosphere," *Tellus*, vol. 14, No. 1, Feb. 1962, pp. 102–115.
2. A. Defant, "Die Zirculation der Atmosphäre in den gemässigten Breiten der Erde," *Geografiske Annaler*, vol. 3, 1921, pp. 209–266.
3. A. Defant, *Physical Oceanography*, vol. 1, Part I, Pergamon Press, 1961, 113 pp.
4. W. C. Jacobs, "Large-Scale Aspects of Energy Transformation Over the Oceans," *Compendium of Meteorology*, American Meteorological Society, Boston, 1951, pp. 1057–1070.
5. H. Landsberg, "Climatology," Section XII of *Handbook of Meteorology*, McGraw-Hill Book Co., Inc., New York, 1945, pp. 927–997 (pp. 949–954).
6. H. Lettau, *Atmosphärische Turbulenz*, Akademische Verlagsgesellschaft, Leipzig, 1939.
7. J. London, "A Study of the Atmospheric Heat Balance," *Final Report* on contract AF19(122)–165, New York University, 1957, 99 pp.
8. Soviet Ministry of Meteorology, "Monthly Charts of the Geopotential and of the Temperature of the Air for 1959," *Bulletin of the Central Meteorological Institute*, Moscow, USSR, 1959.
9. H. U. Sverdrup, "Evaporation from the Oceans," *Compendium of Meteorology*, American Meteorological Society, Boston, 1951, pp. 1071–1081 (p. 1073).
10. U.S. Weather Bureau, "Normal Weather Charts for the Northern Hemisphere," *Technical Paper No. 21*, Washington, D.C., 1952.